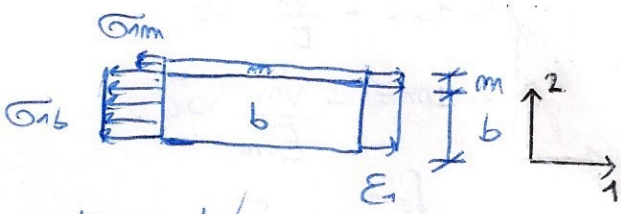
Meccanica della muratura

•) modulo elastico equivalente



$$\begin{cases} \sigma_{1b} = E_b \cdot \varepsilon_1 \\ \sigma_{1m} = E_m \cdot \varepsilon_1 \end{cases}$$

Comportamento medio: $\sigma_1 = \dfrac{\sigma_{1b} \cdot b + \sigma_{1m} \cdot m}{b+m}$

$$\Rightarrow \begin{cases} \eta_b = \dfrac{b}{b+m} \\ \eta_m = \dfrac{m}{b+m} = 1 - \eta_b \end{cases} \Rightarrow \sigma_1 = \sigma_{1b} \cdot \eta_b + \sigma_{1m} \eta_m =$$

$$= (\eta_b \cdot E_b + \eta_m \cdot E_m) \cdot \varepsilon_1$$

$$\Rightarrow E_1 = \eta_b E_b + \eta_m E_m$$

••) legame:

$$\begin{Bmatrix} \varepsilon_{1b} \\ \varepsilon_{2b} \\ \gamma_b \end{Bmatrix} = \begin{bmatrix} 1/E_b & -\nu_b/E_b & 0 \\ -\nu_b/E_b & 1/E_b & 0 \\ 0 & 0 & 1/G_b \end{bmatrix} \begin{Bmatrix} \sigma_{1b} \\ \sigma_{2b} \\ \tau_b \end{Bmatrix} \Rightarrow \begin{cases} \varepsilon_{1b} = \dfrac{\sigma_{1b}}{E_b} - \dfrac{\nu_b}{E_b} \cdot \sigma_{2b} \\ \varepsilon_{2b} = -\dfrac{\nu_b}{E_b} \sigma_{1b} + \dfrac{\sigma_{2b}}{E_b} \\ \gamma_b = \dfrac{\tau_b}{G_b} \end{cases}$$

•••) deformazioni in direzione 1:

$$\varepsilon_2 = \frac{\varepsilon_{2b} \cdot b + \varepsilon_{2m} \cdot m}{b+m} = -\frac{\nu_{12}}{E_1} \cdot \sigma_1$$

$$\Rightarrow \eta_b \varepsilon_{2b} + \eta_m \varepsilon_{2m} = -\frac{\nu_{12}}{E_1} \sigma_1$$

$$\Rightarrow \left[\eta_b \cdot \frac{\nu_b}{E_b} \sigma_{1b} + \eta_m \frac{\nu_m}{E_m} \sigma_{1m} \right] = -\frac{\nu_{12}}{E_1} \sigma_1$$

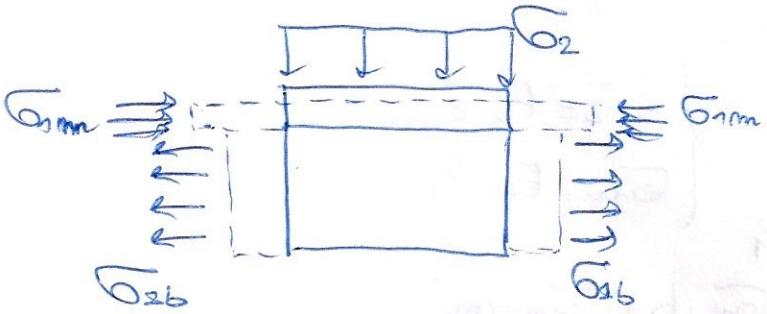
$$\Rightarrow \nu_{12} = \eta_b \cdot \nu_b + \nu_m \cdot \eta_m$$

•••) deformazioni in direzione 2:

$$\varepsilon_2 = \frac{b \cdot \sigma_2/E_b + m \cdot \frac{\sigma_2}{E_m}}{b+m} = \left(\frac{\eta_b}{E_b} + \frac{\eta_m}{E_m} \right) \sigma_2$$

$$\Rightarrow E_2 = \left(\frac{\eta_b}{E_b} + \frac{\eta_m}{E_m} \right)^{-1}$$

Esistono in realtà tensioni interne che equilibrano le differenze tra malta e mattone:



$$\begin{cases} \varepsilon_{1b} = -\frac{\nu_b}{E_b}\sigma_2 \\ \varepsilon_{1m} = -\frac{\nu_m}{E_m}\sigma_2 \end{cases}$$

$$\Rightarrow \begin{cases} \varepsilon_{1b} = \varepsilon_{1m} \\ \sigma_{1m}\, m + \sigma_{1b}\, b = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \sigma_{1m} = -\frac{b}{m}\sigma_{1b} \\ \frac{\sigma_{1b}}{E_b} - \frac{\nu_b}{E_b}\cdot\sigma_2 = -\frac{b}{m}\cdot\frac{\sigma_{1b}}{E_m} - \frac{\nu_m}{E_m}\cdot\sigma_2 \quad (1) \end{cases}$$

$$(1)\quad \frac{\sigma_{1b}}{m}\left(\frac{m}{E_b}+\frac{b}{E_m}\right) = \left(\frac{\nu_b}{E_b}-\frac{\nu_m}{E_m}\right)\sigma_2$$

$$\Rightarrow \frac{mE_m + bE_b}{E_b\cdot E_m}\cdot\frac{b+m}{b+m}\cdot\frac{\sigma_{1b}}{m} = \left(\frac{\nu_b}{E_b}-\frac{\nu_m}{E_m}\right)\sigma_2$$

$$\Rightarrow \underbrace{(\eta_m E_m + \eta_b E_b)}_{=E_1}\frac{b+m}{E_m E_b}\cdot\frac{\sigma_{1b}}{m} = \left(\frac{\nu_b}{E_b}-\frac{\nu_m}{E_m}\right)\sigma_2$$

$$\Rightarrow \sigma_{1b} = \eta_m\frac{E_m E_b}{E_1}\left(\frac{\nu_b}{E_b}-\frac{\nu_m}{E_m}\right)\sigma_2$$

$$\Rightarrow \sigma_{1m} = -\frac{b}{m}\eta_m\frac{E_m E_b}{E_1}\left(\frac{\nu_b}{E_b}-\frac{\nu_m}{E_m}\right)\sigma_2$$

Sostituzione in $\varepsilon_2 = \eta_b\varepsilon_{2b} + \eta_m\varepsilon_{2m}$:

$$\varepsilon_2 = \eta_b\left(-\frac{\nu_b}{E_b}\sigma_{1b} + \frac{\sigma_2}{E_b}\right) + \eta_m\left(-\frac{\nu_m}{E_m}\sigma_{1m} + \frac{\sigma_2}{E_m}\right) =$$

$$= \eta_b\left[-\frac{\nu_b}{E_b}\cdot\eta_m\frac{E_m E_b}{E_1}\left(\frac{\nu_b}{E_b}-\frac{\nu_m}{E_m}\right)\sigma_2 + \frac{\sigma_2}{E_b}\right] +$$

$$+ \eta_m\left[+\frac{\nu_m}{E_m}\frac{b}{m}\eta_m\frac{E_m E_b}{E_1}\left(\frac{\nu_b}{E_b}-\frac{\nu_m}{E_m}\right)\sigma_2 + \frac{\sigma_2}{E_m}\right] =$$

$$= \frac{\eta_b}{E_b} + \frac{\eta_m}{E_m} + \eta_m\eta_b\cdot\frac{E_m E_b}{E_1}\left(\frac{\nu_b}{E_b}-\frac{\nu_m}{E_m}\right)\left(\frac{\nu_m}{E_m}-\frac{\nu_b}{E_b}\right) =$$

$$= \left[\frac{\eta_b}{E_b} + \frac{\eta_m}{E_m} - \eta_m\cdot\eta_b\frac{E_m E_b}{E_1}\left(\frac{\nu_b}{E_b}-\frac{\nu_m}{E_m}\right)^2\right]\sigma_2$$

$$\Rightarrow \varepsilon_2 = \frac{\eta_b}{E_b} + \frac{\eta_m}{E_m} - \eta_m\eta_b\frac{E_m E_b}{E_1}\left(\frac{\nu_b}{E_b}-\frac{\nu_m}{E_m}\right)^2$$