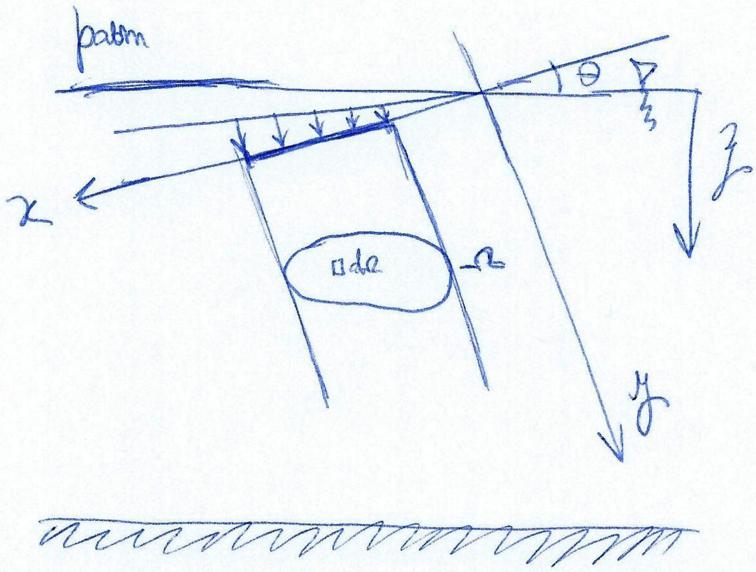
Baricentro e centro di spinta

Superficie sommersa e inclinata:



$$d\underline{S} = -\underline{t}\, d\Omega \qquad \underline{t} = -p\underline{n}$$

$$\Rightarrow d\underline{S} = p\underline{n}\, d\Omega = \gamma z\, \underline{n}\, d\Omega$$

z è l'affondamento

Integrando:

$$\underline{S} = \int_\Omega d\underline{S} = \int_\Omega \gamma z\, \underline{n}\, d\Omega = \underline{n}\int_\Omega \gamma z\, d\Omega = \underline{n}\, S$$

Su una superficie la spinta è diretta normalmente alla superficie stessa. Abbiamo sostituito una spinta equivalente alla spinta infinitesima, ma dobbiamo definirne modulo e punto di applicazione:

$$S = \int_\Omega \gamma z\, d\Omega = \gamma\int_\Omega z\, d\Omega = \gamma\int_\Omega x\,\mathrm{sen}\theta\, d\Omega = \gamma\,\mathrm{sen}\theta\int_\Omega x\, d\Omega$$

Definiamo il baricentro:

$$\begin{cases} x_G = \dfrac{\int_\Omega x\, d\Omega}{\Omega} \\[2ex] y_G = \dfrac{\int_\Omega y\, d\Omega}{\Omega} \end{cases} \Rightarrow S = \gamma\,\mathrm{sen}\theta\, x_G\, \Omega = \gamma z_G\, \Omega = p_G\, \Omega$$

Centro di spinta:

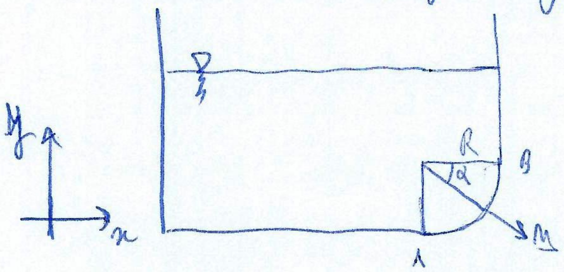
$$\begin{cases} S x_C = \int_\Omega x\, dS \\ S y_C = \int_\Omega y\, dS \end{cases} \Leftrightarrow \begin{cases} x_C = \dfrac{\int_\Omega x\, dS}{S} = \dfrac{\int_\Omega x \gamma z\, d\Omega}{\gamma z_G \Omega} = \dfrac{\int_\Omega \cancel{\gamma} x^2 \cancel{\mathrm{sen}\theta}\, d\Omega}{\cancel{\gamma} x_G \cancel{\mathrm{sen}\theta}\, \Omega} \\[3ex] y_C = \dfrac{\int_\Omega y\, dS}{S} = \dfrac{\int_\Omega y \gamma z\, d\Omega}{\gamma z_G \Omega} = \dfrac{\int_\Omega \cancel{\gamma} xy\, \cancel{\mathrm{sen}\theta}\, d\Omega}{\cancel{\gamma} x_G \cancel{\mathrm{sen}\theta}\, \Omega} \end{cases}$$

$$\Leftrightarrow \begin{cases} x_C = \dfrac{\int_\Omega x^2\, d\Omega}{x_G \Omega} = \dfrac{J_{yy}}{x_G \Omega} = \dfrac{J_{y_0 y_0} + \Omega x_G^2}{x_G \Omega} = x_G + \dfrac{J_{y_0 y_0}}{x_G \Omega} \\[3ex] y_C = \dfrac{\int_\Omega xy\, d\Omega}{x_G \Omega} = \dfrac{J_{xy}}{x_G \Omega} = \dfrac{J_{x_0 y_0} + \Omega x_G y_G}{x_G \Omega} = y_G + \dfrac{J_{x_0 y_0}}{x_G \Omega} \end{cases}$$

Se x_0 o y_0 è di simmetria $J_{x_0 y_0} = 0$ e $G \equiv C$. Si ha sempre $x_C > x_G$. Andando in profondità C si avvicina a G. Per $z \to \infty \Rightarrow C \equiv G$. Se la superficie è parallela al pelo libero $x_C = x_G$.

Superfici gobbe.

Vasca con superficie gobba (larghezza 1 m):



$$\underline{F} = \int_\Omega \underline{t}\, d\Omega = -\int_\Omega p\, \underline{n}\, d\Omega$$

$$\underline{S} = \int_\Omega p\, \underline{n}\, d\Omega \qquad \underline{n} = (\cos\alpha, -\operatorname{sen}\alpha)$$

$$p = \gamma z = \gamma(h + R\operatorname{sen}\alpha)$$

Proiettando su x e y:

$$\begin{cases} S_x = \int_0^{\pi/2} p \cos\alpha\, R\, d\alpha = \gamma R \int_0^{\pi/2} (h + R\operatorname{sen}\alpha)\cos\alpha\, d\alpha \\ S_y = -\int_0^{\pi/2} p \operatorname{sen}\alpha\, R\, d\alpha = -\gamma R \int_0^{\pi/2} (h + R\operatorname{sen}\alpha)\operatorname{sen}\alpha\, d\alpha \end{cases}$$

Ricordando che:

$$\int \cos\alpha\, d\alpha = \operatorname{sen}\alpha + cost \qquad \int \cos\alpha \operatorname{sen}\alpha\, d\alpha = \frac{1}{4} - \frac{1}{4}\cos 2\alpha + cost = -\frac{1}{4}\cos 2\alpha + cost$$

$$\int \operatorname{sen}\alpha\, d\alpha = -\cos\alpha + cost \qquad \int \operatorname{sen}^2\alpha\, d\alpha = \frac{\alpha}{2} - \frac{1}{4}\operatorname{sen} 2\alpha + cost$$

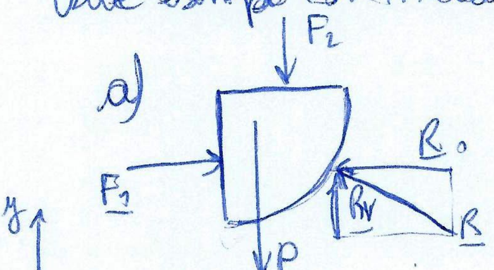
Allora:

$$\begin{cases} S_x = \gamma R \left[h\operatorname{sen}\alpha + R\left(-\frac{1}{4}\cos 2\alpha\right)\right]_0^{\pi/2} = \gamma R\left(h + \frac{R}{2}\right) = \gamma R h + \gamma \frac{R^2}{2} \\ S_y = -\gamma R\left[-h\cos\alpha + R\left(\frac{\alpha}{2} - \frac{1}{4}\operatorname{sen}2\alpha\right)\right]_0^{\pi/2} = -\gamma R\left(h + \frac{R\pi}{4}\right) = -\gamma R h - \gamma\frac{R^2\pi}{4} \end{cases}$$

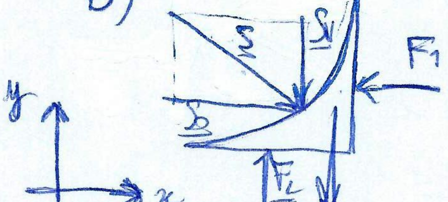
Infine $S = \sqrt{S_x^2 + S_y^2}$ e $tg\,\theta = -\frac{S_y}{S_x}$

Due esempi con metodo semplificato:

a)



b)



a) $\begin{cases} \underline{F_1} - \underline{R_o} = 0 \\ \underline{F_2} + \underline{P} - \underline{R_V} = 0 \end{cases}$

b) $\begin{cases} \underline{S_o} - \underline{F_1} = 0 \\ \underline{S_V} + \underline{P} - \underline{F_2} = 0 \end{cases}$

$$\begin{cases} S_x = -R_x = R_o = F_1 \\ S_y = -R_y = -R_V = -F_2 - P \end{cases} \qquad \begin{cases} S_x = S_o = F_1 \\ S_y = -S_V = P - F_2 \end{cases}$$

$$\Rightarrow \begin{cases} S_x = \gamma R h + \gamma\frac{R^2}{2} \\ S_y = -\gamma R h - \gamma\frac{R^2}{4}\pi \end{cases} \qquad \Rightarrow \begin{cases} S_x = \gamma R h + \gamma\frac{R^2}{2} \\ S_y = \gamma\left(R^2 - \frac{R^2}{4}\pi\right) - (h+R)\gamma R = \\ \quad = -\gamma R h - \gamma\frac{R^2}{4}\pi \end{cases}$$